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Chiral Perturbation Theory and Nuclear Physics *

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Abstract

Chiral symmetry serves as a guiding principle in low-energy hadron dynamics. An effective lagrangian, which explicitly breaks chiral symmetry via a small mass term, allows for a systematic method of calculating higher order corrections to tree-level results. This is applied to radiative muon capture on hydrogen, and to proton-proton π^0 production near threshold. Both processes pose theoretical challenges in light of recent experimental results.

I. INTRODUCTION

One of the more challenging research topics of modern nuclear physics is to relate the strong interaction dynamics of the quarks to the hadronic degrees of freedom of the nucleus. We know that the nucleons consists of at least three valence quarks, and we consider the quark dynamics to be determined by QCD. However, the predictions of QCD in hadronic low energy phenomena have proven too difficult to evaluate. For example, no one has been able to derive from QCD the strong nuclear forces between two nucleons. The analogous QED problem would be to calculate the van der Waals forces between two atoms, a task which has been accomplished successfully. Therefore, to investigate the low-energy strong hadronic interactions, the second best strategy would be to construct QCD inspired models (which all have their flaws), and/or to assume that the symmetry properties of QCD will manifest themselves also on the hadronic level. In this paper I will concentrate on hadronic manifestations and predictions of the almost perfect chiral symmetry of the QCD lagrangian. Provided the quarks are massless the QCD lagrangian is chiral symmetric. In this ideal case the chirality (or helicity) of a quark is conserved, meaning a right- and a left- handed quark will always retain their handedness. A quark mass term in the lagrangian will violate this symmetry and will allow left- and right- handed quarks to mix (or interact via spin-flip). Fortunately the u- and d- quark masses, $m_u \approx 7$ MeV and $m_d \approx 15$ MeV, are small compared to the QCD scale $\Lambda_{QCD} \sim 200$ MeV. It is therefore reasonable to consider the explicit breaking of chiral symmetry via the quark mass term in $\mathcal{L}_{QCD}$ as a small perturbation. (In the following we shall not consider the implied isospin violations which we know is small, but insist that $m_u = m_d = m_q$.) From the Gell-Mann, Oakes and Renner (GOR) relation the pion mass squared: $m_\pi^2 \propto m_q$. This means that the pions are massless in the chiral symmetric limit (when $m_q \neq 0$). Below we shall argue that we can calculate the corrections to the chiral symmetry predictions of hadronic observables in a consistent, perturbative manner, via an approach called chiral perturbation theory (ChPT).

After a short presentation of the essential points of ChPT, two examples of low energy hadronic physics will be presented where ChPT might give some valuable insight. Finally a brief overview of consequences of chiral symmetry considerations as applied to dense nuclear

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matter and the question of possible kaon condensation in neutron stars. It was this last question and its possible consequences in supernova explosion by Brown and Bethe [1] that triggered our investigations of ChPT.

II. CHIRAL SYMMETRY AND THE EFFECTIVE LAGRANGIAN

The seminal work of Gasser and Leutwyler [2] has shown that ChPT is a very powerful and successful technique for gaining more insight of hadronic phenomenology at low energies. Its application includes the physics of the pseudoscalar mesons (π,K,η) and their interactions [3], the pion-nucleon systems [4] and to some extent the few-nucleon systems [5,6].

The effective chiral lagrangian of ChPT $\mathcal{L}_{\rm ch}$ that describes the low energy ($E < \Lambda_\chi \sim 1$ GeV) hadronic phenomena involves an SU(2) matrix $U(x)$ which is non-linearly related to the pion field, e.g., $U(x) = \sqrt{1-[\boldsymbol{\pi}(x)/f_\pi]^2} + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)/f_\pi$.[1] In the meson sector, the sum of chiral-invariant monomials constructed from $U(x)$ and its derivatives constitute the chiral-symmetric part of $\mathcal{L}_{\rm ch}$. The symmetry-breaking part of $\mathcal{L}_{\rm ch}$ is given via a mass matrix $\mathcal{M}$, the chiral transformation of which is dictated by that of the quark mass term in the QCD lagrangian. $\mathcal{L}_{\rm ch}$ is written as an expansion in powers of ∂_μ and the mass matrix $\mathcal{M}$, both characterized by a generic pion momentum $\tilde{Q} \ll \Lambda_\chi$, where $\tilde{Q}$ represents either the pion momentum q or the pion mass m_π. This means each term appearing in $\mathcal{L}_{\rm ch}$ carry a factor $(\tilde{Q}/\Lambda)^{\bar{\nu}}$, where the chiral order index $\bar{\nu}$ is defined by $\bar{\nu} \equiv d-2$, where d is the summed power of the derivative and the pion mass involved in this term. This suggests the possibility of describing low-energy hadronic phenomena in terms of $\mathcal{L}_{\rm ch}$ that contains only a manageably limited number of terms of low chiral order. This is the basic idea of ChPT.

The heavy fermion formalism (HFF) [7] allows us to easily extend ChPT to the meson-nucleon system. This HFF is used since ∂_0 acting on the ordinary Dirac field ψ describing the nucleon yields $\sim M$, a mass which is not small compared with Λ_χ. In HFF, ψ is replaced by the heavy nucleon field $N(x)$ and the accompanying "small field" $n(x)$ through the "transformation" $\psi(x) = \exp(-iMv\cdot x)\,[N(x)+n(x)]$. To obtain these new fields we use the projection operators $\mathcal{P}^v_\pm = (1\pm\not{v})/2$ to project out the "heavy"-, N, and "small"-, n, fields from the Dirac spinor. By definition $N(x) \equiv \mathcal{P}^v_+ exp(iMv\cdot x)\psi(x)$. Here the four-velocity v_μ is assumed to be static, i.e., $v_\mu \sim (1,0,0,0)$. A systematic elimination of the "small" field $n(x)$ in favor of $N(x)$ leads to an expansion in ∂_μ/M. Since $M \approx 1$ GeV $\approx \Lambda_\chi$, an expansion in ∂_μ/M may be treated like the expansion in $\partial_\mu/\Lambda_\chi$. In HFF $\mathcal{L}_{\rm ch}$ consists of chiral symmetric monomials constructed from $U(x)$, $N(x)$ and their derivatives in addition to the symmetry-breaking terms. Including the fermions the chiral order $\bar{\nu}$ in HFF is defined by $\bar{\nu} \equiv d + n/2 - 2$, where d is, as before, the summed power of the derivative and the pion mass, while n is the number of nucleon fields involved in a given term [8]. As before, each term in $\mathcal{L}_{\rm ch}$ carry a factor $(\tilde{Q}/\Lambda)^{\bar{\nu}} \ll 1$. We use $\mathcal{L}_{\rm ch}$ as given in [4].

In the most general form the effective lagrangian $\mathcal{L}_{\rm ch}$ involving pions and heavy nucleons in external weak- and electromagnetic-fields consistent with chiral symmetry, is written in increasing chiral order as:

$$\mathcal{L}_{\rm ch} = \mathcal{L}^{(0)}_\pi + \mathcal{L}^{(0)}_{\pi N} + \mathcal{L}^{(1)}_{\pi N} + \cdots, \qquad (1)$$

Here $\mathcal{L}^{(\bar{\nu})}$ represents terms of chiral order $\bar{\nu}$, and the explicit expressions for the $\mathcal{L}^{(0)}_\pi$, $\mathcal{L}^{(0)}_{\pi N}$ and $\mathcal{L}^{(1)}_{\pi N}$, which only include terms of direct relevance for our calculations, are

[1] Another commonly used parameterization is $U(x) = \exp[i\boldsymbol{\tau}\cdot\boldsymbol{\phi}(x)/f_\pi]$.

$$\mathcal{L}_{\pi}^{(0)} = \frac{f_\pi^2}{4}\mathrm{Tr}\,[D_\mu U D^\mu U] + m_\pi^2(U^\dagger + U - 2)] + \ldots \quad (2)$$

$$\mathcal{L}_{\pi N}^{(0)} = \bar{N}\,\{iv\cdot D + g_A S\cdot u\}\,N - \frac{1}{2}\sum_A C_A(\bar{N}\Gamma_A N)^2 \quad (3)$$

$$\mathcal{L}_{\pi N}^{(1)} = \bar{N}\Big\{\frac{1}{2M}(v\cdot D)^2 - \frac{1}{2M}D\cdot D - \frac{ig_A}{2M}\{S\cdot D, v\cdot u\}_+ + 2c_1 m_\pi^2 \bar{N}N\mathrm{Tr}(U+U^\dagger-2) + (c_2 - \frac{g_A^2}{8M})\bar{N}(v\cdot u)^2 N + c_3\bar{N}u\cdot uN - \frac{i}{4M}[S^\mu, S^\nu]_-\left((1+\kappa_v)f_{\mu\nu}^+ + \frac{1}{2}(\kappa_s - \kappa_v)\mathrm{Tr}f_{\mu\nu}^+\right)\Big\}N + \ldots \quad (4)$$

Above we have used standard notations [4]:

$$D_\mu U \equiv \partial_\mu U - i(\mathcal{V}_\mu + \mathcal{A}_\mu)U + iU(\mathcal{V}_\mu - \mathcal{A}_\mu);$$
$$U \equiv u^2; \qquad u_\mu \equiv iu^\dagger D_\mu U u^\dagger = i\left(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger\right);$$
$$D_\mu N \equiv \partial_\mu N + \frac{1}{2}[u^\dagger, \partial_\mu u]_- N - \frac{i}{2}u^\dagger(\mathcal{V}_\mu + \mathcal{A}_\mu)u - \frac{i}{2}u(\mathcal{V}_\mu - \mathcal{A}_\mu)u^\dagger;$$
$$F_\mu^R \equiv \mathcal{V}_\mu + \mathcal{A}_\mu; \qquad F_\mu^L \equiv \mathcal{V}_\mu - \mathcal{A}_\mu$$
$$F_{\mu\nu}^{L,R} \equiv \partial_\mu F_\nu^{L,R} - \partial_\nu F_\mu^{L,R} - i[F_\mu^{L,R}, F_\nu^{L,R}]_-; \quad f_{\mu\nu}^+ \equiv u^\dagger F_{\mu\nu}^R u + uF_{\mu\nu}^L u^\dagger. \quad (5)$$

The covariant derivatives above include the external vector and axial vector fields, $\mathcal{V}_\mu = \mathcal{V}_\mu^a \frac{\tau^a}{2}$ and $\mathcal{A}_\mu = \mathcal{A}_\mu^a \frac{\tau^a}{2}$, respectively. The covariant spin operator S^μ of the heavy nucleon is defined by $S_\mu \equiv \frac{1}{4}\gamma_5[\not{v}, \gamma_\mu]$, which for our static velocity equals $S^\mu = (0, \frac{1}{2}\vec{\sigma})$. The coupling constants c_1, c_2 and c_3 can be fixed from phenomenology [4]. Their values are related to the pion-nucleon σ-term, $\sigma_{\pi N}(t) \sim \langle p'|m_q(\bar{u}u + \bar{d}d)|p\rangle$ (where $t = (p'-p)^2$), the axial polarizability α_A, and the isospin-even πN s-wave scattering length $a^+ \equiv \frac{1}{3}(a_{1/2} + 2a_{3/2}) \approx -0.008 m_\pi^{-1}$. The four-Fermi non-derivative contact terms in Eq.(3) were introduced by Weinberg [8] and further investigated in two- and three-nucleon systems by van Kolck *et al.* [9]. (In Eq.(3) the sum over A runs over the possible combinations of γ- and $\boldsymbol{\tau}$- matrices: $\Gamma_S^S = 1$, $\Gamma_S^V = \boldsymbol{\tau}$, $\Gamma_V^S = S_\mu$, and $\Gamma_V^V = S_\mu\boldsymbol{\tau}$. However, because of the Fermi statistics (Fierz rearrangement), only two of the four coupling constants C_A are independent.) Although these terms are important in the chiral perturbative derivation of the short range nucleon-nucleon interactions not included in multi-pion exchanges [8,9], they do not play a major role in the following discussion.

In addition to the chiral order index $\bar{\nu}$ defined for each term in $\mathcal{L}_{\rm ch}$, a chiral order index ν is assigned for each irreducible Feynman diagram appearing in the chiral perturbation series [8]. Its definition for a multifermion system is

$$\nu = 4 - E_N - 2C + 2L + \sum_i \bar{\nu}_i, \quad (6)$$

where E_N is the number of nucleons in the Feynman diagram, L the number of loops, and C the number of disconnected parts of the diagram. The sum over i runs over all the vertices in the Feynman graph, and $\bar{\nu}_i$ is the chiral order of each vertex. One can show [8] that an irreducible diagram of chiral order ν carries a factor $(\tilde{Q}/\Lambda)^\nu \ll 1$. However, the application of ChPT to nuclei involves some subtlety. As emphasized by Weinberg [8], naive chiral counting fails for a nucleus, which is a loosely bound many-body system. This is because purely nucleonic intermediate states occurring in a nucleus can have very low excitation energies, which spoils the ordinary chiral counting. To avoid this difficulty, one must first

classify diagrams appearing in perturbation series into irreducible and reducible diagrams, according to whether or not a diagram is free from purely nucleonic intermediate states. Thus, in an irreducible diagram, every intermediate state contains at least one meson. The ChPT can be applied to the irreducible diagrams. The contribution of all the irreducible diagrams (up to a specified chiral order) is then to be used as an effective operator acting on the nucleonic Hilbert space. This second step allows us to incorporate the contributions of the reducible diagrams. We may refer to this two-step procedure as the *nuclear chiral perturbation theory* (nuclear ChPT). This method was first applied by Weinberg [8] and by van Kolck *et al.* [9] to the few nucleon system. Park, Min and Rho [6] applied the nuclear ChPT to meson exchange currents in nuclei, and others had success in describing the exchange currents for the electromagnetic and weak interactions [10,11].

In the literature the term "effective lagrangian" is often used to imply that the lagrangian is only meant for calculating tree diagrams. We must note, however, that the "modern" effective lagrangians have a different meaning. Not only can $\mathcal{L}_{\rm ch}$ of Eq.(1) be used beyond tree approximation but, in fact, a consistent chiral counting demands inclusion of every loop diagram whose chiral order ν is lower than or equal to the chiral order of interest. As will be discussed below, for a consistent ChPT treatment of the problem at hand, we therefore need to consider loop corrections and necessary lagrangian counterterms of chiral order ν to cancel the loop-divergences.

III. CHPT APPLICATIONS

A. Radiative muon capture

The radiative muon capture on the proton (RMC) $\mu^- + p \to n + \nu_\mu + \gamma$ has an extremely small branching ratio and to observe RMC is a great experimental challenge. Only recently has an experimental group at TRIUMF [12] finally succeeded in measuring the partial capture rate $\Gamma_{\rm RMC}(> 60{\rm MeV})$, corresponding to emission of a photon with $E_\gamma > 60{\rm MeV}$. A main goal of measuring $\Gamma_{\rm RMC}$ is to extract accurate information about the pseudoscalar form factor, f_P, of the weak hadronic matrix element. The matrix element of the hadronic charged weak current $h^\lambda = V^\lambda - A^\lambda$ between a proton and a neutron is

$$\begin{aligned}\langle n(p_f)|V^\lambda - A^\lambda|p(p_i)\rangle \;=& \\ \bar{u}(p_f)\left[f_V(q^2)\gamma^\lambda + \frac{f_M(q^2)}{2m_N}\sigma^{\lambda\mu}q_\mu + f_A(q^2)\gamma^\lambda\gamma_5 + \frac{f_P(q^2)}{m_\pi}q^\lambda\gamma_5\right]u(p_i),& \qquad (7)\end{aligned}$$

where $q \equiv p_i - p_f$, and the absence of the second-class current is assumed. Ordinary muon capture on a proton (OMC), $\mu^- + p \to n + \nu_\mu$, gives only an approximate value for f_P. The reason is that the momentum transfer in OMC, $q^2 = -0.88m_\mu^2$, is far away from the pion-pole position, $q^2 = m_\pi^2$, where the contribution of $f_P(q^2)$ becomes most important. RMC provides a more sensitive probe of f_P because in the RMC's three-body final state one can be closer to the pion-pole. Using the theoretical framework of Beder and Fearing [13] the TRIUMF group extracted a value for f_P about 50% larger than expected from PCAC. This surprising result should be contrasted with the fact that f_P measured in OMC is consistent with the PCAC prediction within large experimental uncertainties. In this framework, as in many earlier works [14,15], one invokes a minimal substitution to generate the RMC transition amplitude from the transition amplitude for OMC, the hadronic part of which is given by Eq. (7). Essentially, one writes $f_P(q^2) = \tilde{f}_P/(q^2 - m_\pi^2)$, where $\tilde{f}_P$ is a constant and then one replaces every q in Eq.(7) with $q - e\mathcal{A}$ ($\mathcal{A}$ is the electromagnetic field) except the q appearing in the q^2 dependence of f_V, f_A and f_M. Due to the large discrepancy with the expected $\tilde{f}_P$ value, it seems reasonable to reexamine the reliability of this existing minimal substitution phenomenological approach [13].

FIGURES

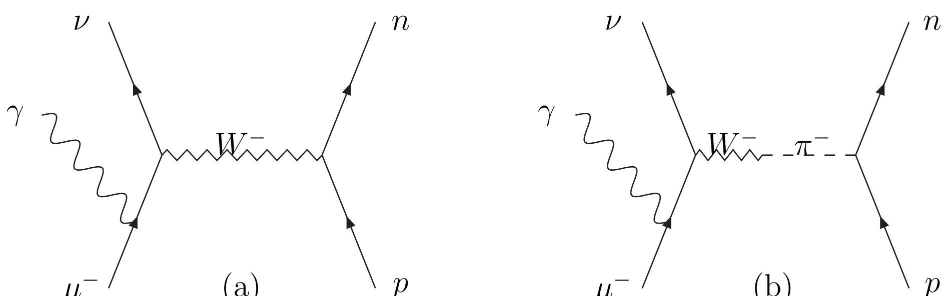

FIG. 1.

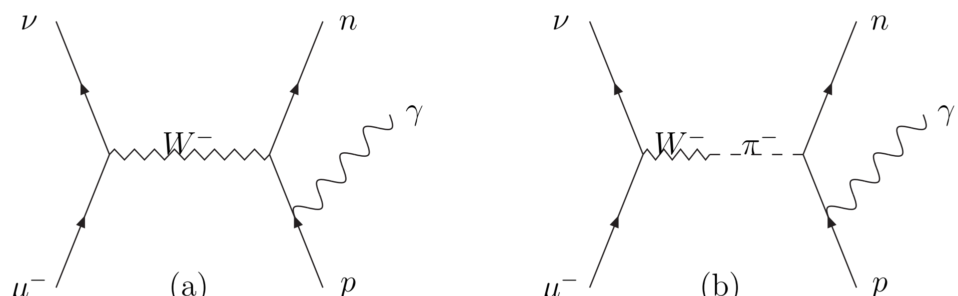

FIG. 2.

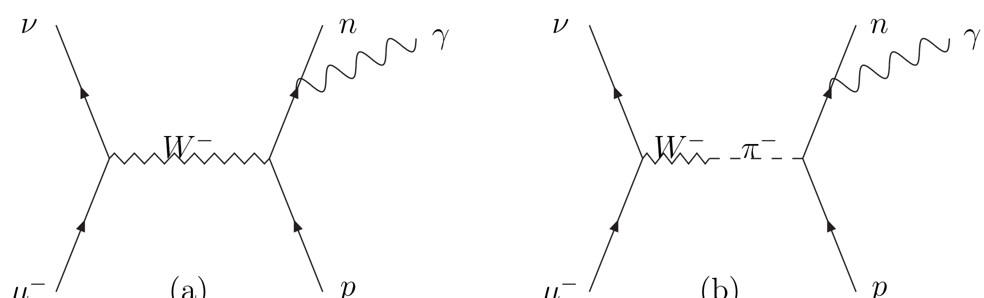

FIG. 3.

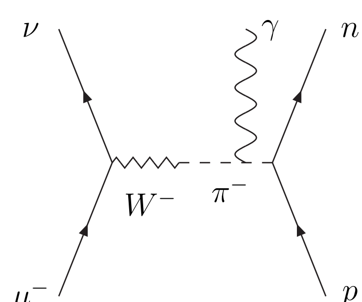

FIG. 4.

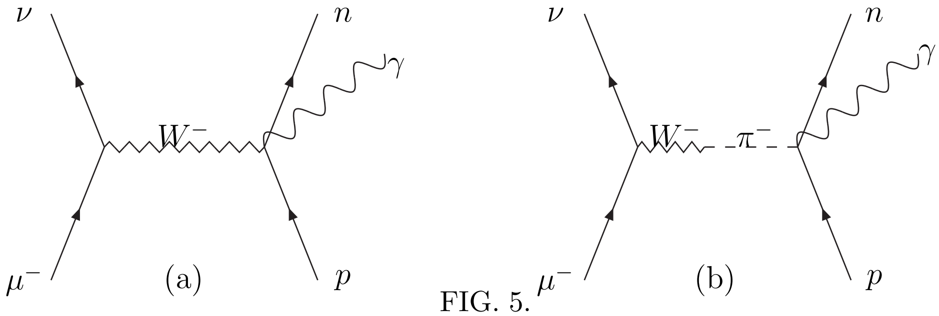

FIG. 5.

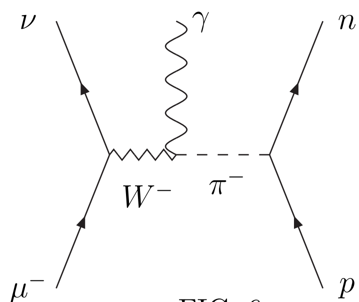

FIG. 6.

ChPT provides a systematic framework to describe the electromagnetic-, weak-, and strong-interaction vertices in a consistent manner, avoiding the phenomenological minimal-coupling substitution at the level of the transition amplitude. Furthermore, ChPT also satisfies the gauge-invariance and chiral-symmetry requirements in a transparent way. In the case of OMC, Bernard et al. [16] and Fearing et al. [17] used heavy-baryon ChPT to evaluate the value of $\tilde{f}_P$ with better accuracy but consistent with the value achieved in the PCAC approach. Muon capture is a favorable case for applying ChPT since momentum transfers involved here do not exceed m_μ, and m_μ is small compared to the chiral scale $\Lambda \sim$ 1 GeV, implying a reasonably rapid convergence of chiral expansion.

A first calculation of the total capture rate $\Gamma_{\rm RMC}$ and the spectrum of the emitted photons, $d\Gamma_{\rm RMC}(k)/dk$, in chiral perturbation expansion will be briefly discussed [18]. We limit ourselves here to a next-to-leading chiral order (NLO) calculation and therefore we only consider the terms with $\bar{\nu} = 0$ and $\bar{\nu} = 1$ in Eq.(1). To this chiral order we need only consider tree diagrams, and then $\mathcal{L}_{\pi N}^{(1)}$ only represents $1/M$ "nucleon recoil" corrections to the leading "static" part $\mathcal{L}_{\pi N}^{(0)}$. Furthermore, we do not consider explicit Δ degrees of freedom. The covariant derivatives of Eq.(5) include the external fields, $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$. These are determined by the W^- boson which couples to the leptonic and hadronic currents in the standard manner. In the actual calculation, taking the limit $m_W \to \infty$, we make the substitution $W_\mu^- \to (\mathcal{V}_\mu^- - \mathcal{A}_\mu^-)\frac{\tau^1 - i\tau^2}{2}$, i.e. we treat $\mathcal{V}$ and $\mathcal{A}$ as static external sources. The only parameters appearing in the expressions for Eq.(1) relevant for RMC [Eq.(2), first term of Eq.(3) and first and last line of Eq.(4)] are the pion decay constant, $f_\pi = 93$ MeV, the axial vector coupling, $g_A = 1.26$, and the nucleon isoscalar and isovector anomalous magnetic moments, $\kappa_s = -0.12$ and $\kappa_v = 3.71$. Thus, to the chiral order of our interest $\mathcal{L}_{\rm ch}$ is well determined.

We consider all possible Feynman diagrams Figs.1-6 up to chiral order $\nu = 1$ which contribute to the process $\mu^- + p \to n + \nu + \gamma$. The zigzag lines in these diagrams represent the W^- boson. For static W^- bosons the diagrams in Figs.1-6 reduce to those that

would result from the simple current-current interaction of the $V-A$ form. The reason for explicitly retaining the W^- boson lines is to clearly separate the different photon vertices (see e.g. Fig.6). The leptonic vertices in these Feynman diagrams are of course well known. The hadronic vertices are obtained by expanding the ChPT lagrangian [Eqs. (1), (2), (3) and the relevant parts of (4)] in terms of the elementary fields N, π, $\mathcal{V}$ and $\mathcal{A}$ and their derivatives. The evaluation of the transition amplitudes corresponding to these Feynman diagrams is straightforward. We denote by M_i ($i = 1\ldots 6$) the invariant transition amplitudes corresponding to Fig.(1)-(6), respectively. They are calculated in the Coulomb gauge, i.e. $v\cdot\epsilon(\lambda) = 0$, and are given by:

$$\begin{aligned} M_1 &= \epsilon^\beta(\lambda)\left[\bar{u}_\nu(s)\gamma_\tau(1-\gamma_5)\frac{\not{\mu}-\not{k}+m_\mu}{2(k\cdot\mu)}\gamma_\beta u_\mu(s')\right]\left[N_n^\dagger(\sigma)h_1^\tau N_p(\sigma')\right] \qquad (8)\\ M_i &= \left[\bar{u}_\nu(s)\gamma_\tau(1-\gamma_5)u_\mu(s')\right]\left[N_n^\dagger(\sigma)h_i^\tau(\lambda)N_p(\sigma')\right],\ i=2,3,4,5,6 \end{aligned}$$

where h_i^τ; $i = 1,\cdots,6$ are the hadronic operators given in Ref. [18]. In Eqs.(8) and (9) μ, ν, $p = (E_p,\vec{p})$, $n = (E_n,\vec{n})$ and $k = (\omega_k,\vec{k})$ are the four-momenta of the muon, neutrino, proton, neutron and photon, respectively. The z-components of the spins of the muon, neutrino, proton and neutron are denoted by s, s', σ' and σ, respectively, while $\epsilon(\lambda)$ stands for the photon polarization vector.

We consider only the case of RMC from the μ-p atom with statistical spin distributions, leaving out the hyperfine-state decomposition and the treatment of RMC from the $p\mu p$ molecule. With our kinematical approximations: both the muon and the proton are at rest and neglecting the recoil neutron kinetic energy, the spin-averaged total capture rate is

$$\begin{aligned}\Gamma_{\rm RMC} &= \left(\frac{eG}{\sqrt{2}}\right)^2\frac{|\Phi(0)|^2}{4}(2\pi)^4\int\frac{d^3n}{(2\pi)^3}\int\frac{d^3\nu}{(2\pi)^3}\int\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k}\\ &\quad\times\delta^{(4)}(n+\nu+k-p-\mu)\sum_{\sigma\sigma' ss'\lambda}|M|^2 \qquad (9)\end{aligned}$$

where the sum is over all spin and polarization orientations, $M = \sum_{i=1}^6 M_i$, and the μp atomic wavefunction at the origin is $\Phi(0)$. Within the kinematical approximation stated earlier, $\delta^{(4)}(n+\nu+k-p-\mu) \approx \delta(|\vec{k}|+|\vec{\nu}|-m_\mu)\delta^{(3)}(\vec{n}+\vec{\nu}+\vec{k})$ and the maximal γ energy is $(\omega_k)_{\rm max} \approx m_\mu$. The numerical values for the total capture rate $\Gamma_{\rm RMC}$ are then [18]:

1. To lowest chiral order $\nu = 0$ $\Gamma_{RMC} = 0.061\ s^{-1}$ (excluding the pion-pole diagrams' contributions $\Gamma_{RMC} = 0.043\ s^{-1}$).

2. Including the $\nu = 1$ ($\mathcal{O}(1/M)$ recoil terms) contributions give $\Gamma_{RMC} = 0.075\ s^{-1}$ (no pion-pole $= 0.053\ s^{-1}$)

Our total capture rate $\Gamma_{\rm RMC} = 0.075 s^{-1}$ is close to the value given in [14], $\Gamma_{\rm RMC} = 0.069 s^{-1}$, and practically identical to $\Gamma_{\rm RMC} = 0.076 s^{-1}$ reported in [15]. Our $\mathcal{O}(1/M)$ recoil corrections account for about 20% of the leading order $\mathcal{O}((1/M)^0)$ contribution, which indicates a reasonable convergence of the chiral expansion. As one can see about 30% of the total value of $\Gamma_{\rm RMC}$ comes from the pion-pole exchange diagrams in our calculation.

A direct comparison of our calculation with the experimental data [12] is premature because we have not considered captures from the singlet and triplet hyperfine states separately, or capture from the $p\mu p$ molecular state. This also means that at this stage we cannot directly address the "f_P problem" that arose from the TRIUMF data [12]. However, as already stated ChPT gives a unique prediction on f_P, are consistent with the Goldberger-Treiman value $f_P = 6.6 g_A$ [16,17]. Meanwhile, for the spin-averaged μp-atomic RMC, our ChPT calculation gives a photon spectrum that is harder than that of [13] for the PCAC value of f_P. Of course, a more quantitative statement can be made only after a more detailed ChPT calculation becomes available in which the hyperfine states are separated

and the $p\mu p$-molecular absorption is evaluated. We must also emphasize that the present calculation includes only up to the next-to-leading chiral order (NLO) contributions. A next-to-next-to-leading order (NNLO) calculation includes the $\nu = 2$ chiral lagrangian, $\mathcal{L}_{\pi}^{(2)}$ and $\mathcal{L}_{\pi N}^{(2)}$, and also loop corrections arising from $\mathcal{L}_{\pi}^{(0)}$ and $\mathcal{L}_{\pi N}^{(0)}$ has been completed and these next order terms are found to give small corrections to the tree diagrams [19]. (The finite contributions from the loop diagrams would give momentum-dependent vertices, which would correspond to the form factors in the language of the phenomenological approach [13,15].) We note that the approach of Bernard et al. [4], which we have used here, does not contain the explicit Δ degree of freedom in contrast to [20]. For a complete calculation the Δ degrees of freedom should be included in the next chiral order as done by, e.g., Fearing et al. [17].

B. Pion production at threshold

The second application of ChPT is concerned with recent high-precision measurements of the total cross sections near threshold for the reaction $p + p \to p + p + \pi^0$ by Meyer et al. [21]. Traditionally the pion production reactions are described by the single nucleon process (the Born term), Fig.7(a), and the s-wave pion rescattering process, Fig.7(b).

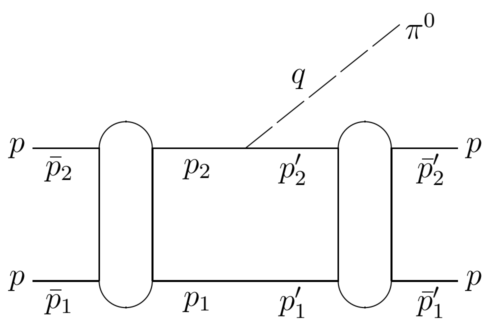

fig. 7(a)

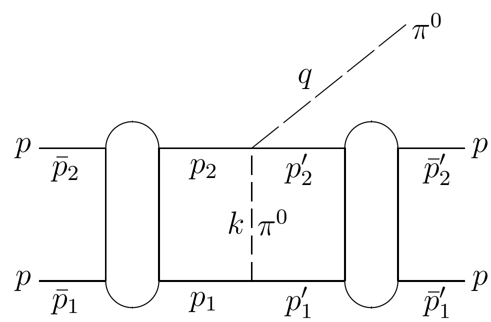

fig. 7(b)

The Born term is assumed to be given by the pseudovector interaction Hamiltonian

$$\mathcal{H}_0 = \frac{g_A}{2f_\pi}\bar{\psi}\left(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}(\boldsymbol{\tau}\cdot\boldsymbol{\pi}) - \frac{i}{2M}\{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}, \boldsymbol{\tau}\cdot\dot{\boldsymbol{\pi}}\}\right)\psi, \tag{10}$$

where g_A is the axial coupling constant, and $f_\pi = 93$ MeV is the pion decay constant. The first term represents p-wave pion-nucleon coupling, while the second term accounts for the nucleon recoil effect and s-wave pion coupling. The s-wave rescattering vertex in Fig.7(b) is commonly calculated using the phenomenological Hamiltonian [22]

$$\mathcal{H}_1 = 4\pi\frac{\lambda_1}{m_\pi}\bar{\psi}\boldsymbol{\pi}\cdot\boldsymbol{\pi}\psi + 4\pi\frac{\lambda_2}{m_\pi^2}\bar{\psi}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}\psi \tag{11}$$

The two coupling constants λ_1 and λ_2 determined from s-wave pion nucleon scattering lengths have the values $\lambda_1 \sim 0.005$ and $\lambda_2 \sim 0.05$. Thus, $\lambda_1 \ll \lambda_2$ as expected from current algebra. The early calculations [22–24] which are based on these phenomenological vertices, underestimate the measured π^0 production cross sections by a factor of $\sim$5. Since the second term in Eq.(10) is suppressed compared to the first term by $\sim m_\pi/M$, the importance of the rescattering term is enhanced. However, λ_2 is much larger than λ_1, and the isospin structure of the λ_2 term is such that it cannot contribute to the π^0 production from two protons at the rescattering vertex in Fig.7(b). Thus, the use of the phenomenological

Hamiltonians, Eqs.(10) and (11), to calculate the Born term and the rescattering terms, gives significantly suppressed cross sections for the $pp \to pp\pi^0$ reaction near threshold compared to π^+ production because only the small correction terms contributes. Therefore, the calculated cross sections can be highly sensitive to any deviations from this conventional treatment.

It is convenient for our discussion to introduce the *typical threshold kinematics* for this reaction. Consider Fig.7(b) in the center-of-mass system with the initial and final interactions turned off (since it will modify what follows). At threshold, $(q_0, \boldsymbol{q}) = (m_\pi, 0)$, $p'_{10} = p'_{20} = M$, and $\boldsymbol{p}'_1 = \boldsymbol{p}'_2 = 0$, so that any exchanged particle must have $k_0 = m_\pi/2 = 70$ MeV and $|\boldsymbol{k}| = \sqrt{m_\pi M + (m_\pi/2)^2} \sim 370$ MeV/c, which implies $k^2 = -m_\pi M$. Thus the rescattering process probes internucleon distances ~ 0.5 fm. The reaction is therefore sensitive to exchange of heavy mesons which are important in phenomenological N-N potentials. Lee and Riska [25] showed that heavy-meson exchanges (scalar and vector exchange) could be capable of enhancing the cross section significantly. Meanwhile, Hernández and Oset [26] and Hanhart *et al.* [27] showed that the *off-shell* dependence of the πN s-wave isoscalar amplitude featuring in the rescattering process, $k^2 = -m_\pi M \neq m_\pi^2$, could also appreciably enhance the rescattering amplitude. Given these developments we consider it important to examine the significance of these phenomenological lagrangians in ChPT.

Systematic studies based on ChPT would be valuable to sharpen our conclusions on whether or not the heavy-meson exchange contributions are needed to explain the observed cross section for $pp \to pp\pi^0$. Three very similar ChPT investigations have recently been completed [28–30]. Here we describe briefly the work of the USC group. To generate the one- and two-body diagrams of Figs. 7(a) and 7(b) we minimally need terms with $\bar\nu = 0$ and 1. Eq.(1) leads to the pion-nucleon interaction Hamiltonian $\mathcal{H}_{int} = \mathcal{H}_{int}^{(0)} + \mathcal{H}_{int}^{(1)}$, with

$$\mathcal{H}_{int}^{(0)} = \frac{g_A}{2f_\pi}\bar N[\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}(\boldsymbol{\tau}\cdot\boldsymbol{\pi})]N + \frac{1}{4f_\pi^2}\bar N\boldsymbol{\tau}\cdot\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}N \tag{12a}$$

$$\mathcal{H}_{int}^{(1)} = \frac{-ig_A}{4m_N f_\pi}\bar N\{\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}, \boldsymbol{\tau}\cdot\dot{\boldsymbol{\pi}}\}N + \frac{1}{f_\pi^2}[2c_1 m_\pi^2\pi^2 - (c_2 - \frac{g_A^2}{8m_N})\dot\pi^2 - c_3(\partial\pi)^2]\bar N N \tag{12b}$$

Here $\mathcal{H}_{int}^{(\bar\nu)}$ represents the term of chiral order $\bar\nu$. We now compare $\mathcal{H}_{int}$ resulting from ChPT, Eq.(12) with the phenomenological effective Hamiltonian $\mathcal{H}_0 + \mathcal{H}_1$, Eqs.(10) and (11). We note that the first term in $\mathcal{H}^{(0)}$ and the first term in $\mathcal{H}^{(1)}$ exactly reproduces $\mathcal{H}_0$. Thus the so-called "Galilean-invariance" term naturally arises as a $1/M$ correction term in HFF. As for the $\pi\pi NN$ vertex, we can associate the second term in $\mathcal{H}_{int}^{(0)}$ to the λ_2 term in $\mathcal{H}_1$, and second term in $\mathcal{H}_{int}^{(1)}$ to the λ_1 term in $\mathcal{H}_1$. Consequently,

$$4\pi\lambda_1^\chi/m_\pi \equiv \tfrac{m_\pi^2}{f_\pi^2}[2c_1 - (c_2 - \tfrac{g_A^2}{8m_N})\tfrac{\omega_q\omega_k}{m_\pi^2} - c_3\tfrac{q\cdot k}{m_\pi^2}] \equiv \kappa(k,q) \tag{13}$$

with $q = (\omega_q, \mathbf{q})$ and $k = (\omega_k, \mathbf{k})$. Now, for *on-shell* low energy pion-nucleon scattering, *i.e.*, $k \sim q \sim (m_\pi, \mathbf{0})$, we equate

$$4\pi\lambda_1^\chi/m_\pi = \kappa_0 \;\equiv\; \kappa(k{=}(m_\pi,\mathbf{0}), q{=}(m_\pi,\mathbf{0})), \tag{14}$$

where

$$\kappa_0 = \frac{m_\pi^2}{f_\pi^2}\left(2c_1 - c_2 - c_3 + \frac{g_A^2}{8M}\right) = -2\pi\left(1 + \frac{m_\pi}{m_N}\right)a^+ + \frac{3g_A^2}{128\pi}\frac{m_\pi^3}{f_\pi^4}. \tag{15}$$

The above cited empirical value for a^+ leads to $\kappa_0 = (0.87 \pm 0.20)\,\mathrm{GeV}^{-1}$. If we keep only the lowest chiral order, the first term in Eq.(15), λ_1^χ is

$$\frac{4\pi\lambda_1^\chi}{m_\pi} \;=\; -2\pi\left(1+\frac{m_\pi}{m_N}\right)a^+ \;=\; (0.43\pm0.20)\,\mathrm{GeV}^{-1}, \tag{16}$$

or $\lambda_1^\chi \;=\; 0.005\pm0.002$, identical to the "standard value" of λ_1 used in the literature, Eq.(11). We note that the ChPT value for $\kappa_0 = 0.87$ GeV^{-1}, Eq.(15), include next chiral order terms $\propto g_A^2$, and is twice the value of the first term, Eq.(16). In our comparison between the traditional and the ChPT approaches below, we shall use Eq.(13). Obviously, since $\kappa(k,q)$ depends on the four-momenta q and k, we cannot identify κ with the constant λ_1. To illustrate how the q and k dependencies in $\kappa(k,q)$ affect the rescattering amplitude of Fig.7(b), we consider the *typical threshold kinematics*, $q\sim(m_\pi,\mathbf{0})$ and $k\sim(\frac{1}{2}m_\pi,\sqrt{m_\pi M})$, and denote this $\kappa(k,q)$ value by κ_{th}.

$$\kappa_{th} = \frac{m_\pi^2}{f_\pi^2}\left[2c_1 - \frac{1}{2}\left(c_2 - \frac{g_A^2}{8m_N}\right) - \frac{c_3}{2}\right] \sim -1.5\mathrm{GeV}^{-1}. \tag{17}$$

Thus the s-wave pion-nucleon interaction is *not only* much stronger than the on-shell cases, of Eqs.(15) and (16), *but* the sign of the off-shell coupling strength is *opposite* to the on-shell cases. The first feature is qualitatively in line with the observation of Refs. [26,27] that the rescattering term should be larger than previously considered. However, the sign of the typical off-shell coupling, κ_{th}, is opposite to the one used in Refs. [26,27]. This flip of the sign drastically changes the pattern of interplay between the Born and rescattering terms. The sign change in ChPT arises from the πN rescattering vertex where the πN *scattering occurs at an energy below the* πN *threshold*. If we force the exchanged pion on the mass-shell and only consider the exchanged pion three-momentum $|\boldsymbol{k}|$ to be off-shell, we essentially recover the results of Hanhart *et al.* [27].

The two-nucleon transition matrix element T for the $pp \to pp\pi^0$ process is $T = \langle\Phi_f|\mathcal{T}|\Phi_i\rangle$, where $|\Phi_i\rangle$ ($|\Phi_f\rangle$) is the initial (final) two-nucleon state distorted by the initial-state (final-state) interactions. Here $\mathcal{T}$ represent the contributions of all irreducible diagrams (up to a specified chiral order ν) for the $pp \to pp\pi^0$ process, and we use $\mathcal{T}$ as an effective transition operator in the Hilbert space of nuclear wavefunctions.

$$\mathcal{T} \;=\; \mathcal{T}^{(-1)} + \mathcal{T}^{(1)} \;\equiv\; \mathcal{T}_{-1}^{\mathrm{Born}} + \mathcal{T}_{+1}^{\mathrm{Res}} \tag{18a}$$

$$\mathcal{T}_{-1}^{\mathrm{Born}} = \frac{g_A}{4m_N f_\pi}\omega_q \sum_{i=1,2} \boldsymbol{\sigma}_i\cdot(\boldsymbol{p}_i' + \boldsymbol{p}_i)\tau_i^0, \tag{18b}$$

$$\mathcal{T}_{+1}^{\mathrm{Res}} = -\frac{g_A}{f_\pi}\sum_{i=1,2}\kappa(k_i,q)\frac{\boldsymbol{\sigma}_i\cdot\boldsymbol{k}_i\tau_i^0}{k_i^2 - m_\pi^2 + i\varepsilon} \tag{18c}$$

The operator $\mathcal{T}^{(\nu)}$ represents the contribution from Feynman diagrams of chiral order ν. The initial and final three- momenta of the i-th proton are $\boldsymbol{p}_i$ and $\boldsymbol{p'}_i$, $k_i \equiv p_i - p_i'$; and $\kappa(k_i,q)$ is as defined in Eq.(13). To chiral order $\nu = 1$ we have additional contributions due to the loop corrections. These loop corrections to the Born diagram, Fig.7(a) introduce an effective form factor at the vertex defined by $\mathcal{L}_{ch}$. The loop corrections do not change drastically our results. Since we cannot reproduce the measured cross section, we ignore these corrections in this paper.

A formally "consistent" treatment of T would consist in using for $|\Phi_i\rangle$ and $|\Phi_f\rangle$ two-nucleon wave functions generated by irreducible diagrams of order up to $\nu = 1$. A problem in this "consistent" ChPT approach is that the intermediate two-nucleon propagators in Fig.7 can be significantly off-mass-shell. Another more practical problem is that, if we include

the initial- and final- two-nucleon (N-N) interactions in diagrams up to chiral order $\nu = 1$, these N-N interactions are not realistic enough to reproduce the known N-N observables. A pragmatic remedy for these problems is to use a phenomenological N-N potential to generate the distorted N-N wavefunctions. Park, Min and Rho [11] used this hybrid approach to study the exchange-current in the $n+p \rightarrow \gamma+d$ reaction and at least, for the low-momentum transfer process studied in Ref. [11], the hybrid method is known to work extremely well.

Following standard practice in nuclear physics the T-matrix was evaluated using r-space wave functions after Fourier transforming $\mathcal{T}_{-1}^{\rm Born}$ and $\mathcal{T}_{+1}^{\rm Res}$, Eq.(18), into r-space. To simplify the Fourier transformation the *fixed kinematics approximation*, $\kappa(k,q) = \kappa_{th}$, Eq.(17), was used [28,29]. The results of these two groups indicate that, for the various nuclear distortion potentials considered, the calculated cross section is much too small to reproduce the experimental cross section. If we define the discrepancy ratio R by $R \equiv \sigma_{tot}^{exp}/\sigma_{tot}^{calc}$, with σ_{tot}^{exp} taken from Ref. [21], then $R \cong 80$ ($R \cong 210$) for the Hamada-Johnston (Reid soft-core) potential, and R happens to be almost constant for the whole range of $E_f \leq 23$ MeV for which σ_{tot}^{exp} is known [29]. Thus, the use of κ_{th}, Eq.(17), results in a significant cancellation between the Born- and the rescattering terms. This destructive interference is in sharp contrast to the constructive interference of Refs. [22,25–27]. However, using ChPT in a systematic fashion we have shown that the contribution of the pion rescattering term can be much larger than obtained in the traditional phenomenological calculations.

We also learn that the *fixed kinematics approximation* (which is commonly used in the literature) should be avoided. There are at least two reasons why this is not a good approximation for this reaction: (i) The initial- and final-state interactions play an essential role in the near-threshold $pp \rightarrow pp\pi^0$ reaction; (ii) The theoretical cross section within the framework of the Born plus rescattering terms is likely to depend on the delicate cancellation between these two terms. In a just completed momentum space calculation we avoid the $\kappa(k,q) = \kappa_{th}$ and the *fixed kinematics approximation* [30]. We use Eq.(18) directly where $\kappa(k,q)$ is given by Eq.(13). Our results show that the magnitude of $\mathcal{T}_{+1}^{\rm Res}$ increases by a factor ~ 3 which means the πN rescattering term, Fig.7(b) dominates [30]. Further, we confirm the sign of the rescattering amplitude found in the first two ChPT calculations [28,29], and we find that the c_1 term in $\kappa(k,q)$ dominates since the πN rescattering terms c_2 and c_3, Eq.(13) average to a tiny value in the distorted wave integrals due to their energy dependences. The constant c_1 is given by the πN sigma term, $\Sigma_{\pi N}(0)$, and the uncertainty in the magnitude of the rescattering amplitude, $T_{+1}^{\rm Res}$, is given directly by the uncertainty in the value of $\Sigma_{\pi N}(0)$. Due to the destructive interference with the Born term, this uncertainty is enhanced in the resulting cross section. However, our calculated cross section is still too small. Presently we [31] are evaluating the next chiral order two-pion exchange diagrams which simulate partially the σ-meson exchange used in Ref. [25]. To achieve a large cross section these terms should be important and this call into question the convergence of ChPT. Phenomenologically one could also include the repulsive ω-exchange when we compare with experiments. In the effective $\mathcal{L}_{ch}$ of Eq.(3) this is included in the four-nucleon terms, where we would calculate the constant C_A as given by ω exchange. For charged pion production from pp, the higher chiral order terms considered here are smaller corrections to the dominant Born- and rescattering terms. For π^0 production, however, these next order chiral correction terms give the dominant amplitudes.

Finally, we should mention that when ChPT is extended to SU$_F(3)$ and we assume the KN sigma term $\Sigma_{KN}(0) \sim c_1$ to dominate, the effective Kaon mass in nuclear matter could become very small and we could have a Kaon-condensate in dense nuclear matter. However, if we use current algebra, "weak" PCAC and standard nuclear physics treatment of meson self-energy in matter, we do not find a Kaon condensate in matter. This is still an open question and the existence of a Kaon condensate in dense nuclear matter opens exciting possibilities in astrophysics, e.g. Ref. [1].

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